Published by Institute of Physics Publishing for SISSA
Received: *January 10, 2005*
Accepted: *May 23, 2005*
Published: *June 28, 2005*

Overproduction of cosmic superstrings

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Abstract: We show that the naive application of the Kibble mechanism seriously underestimates the initial density of cosmic superstrings that can be formed during the annihilation of D-branes in the early universe, as in models of brane-antibrane inflation. We study the formation of defects in effective field theories of the string theory tachyon both analytically, by solving the equation of motion of the tachyon field near the core of the defect, and numerically, by evolving the tachyon field on a lattice. We find that defects generically form with correlation lengths of order M_s^{-1} rather than H^{-1}. Hence, defects localized in extra dimensions may be formed at the end of inflation. This implies that brane-antibrane inflation models where inflation is driven by branes which wrap the compact manifold may have problems with overclosure by cosmological relics, such as domain walls and monopoles.

Keywords: Solitons Monopoles and Instantons, Tachyon Condensation, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Although it is notoriously difficult to test string theory in the laboratory, exciting progress has been made on the cosmological front through the realization that superstrings could appear as cosmic strings within recent popular scenarios for brane-antibrane inflation [1]-[3]. Gravity wave detectors and pulsar timing measurements could thus give the first positive experimental signals coming from superstrings [4] (see, however, [5]). If seen, it will be challenging to distinguish cosmic superstrings from conventional cosmic strings. One distinguishing feature is the possibility that superstrings have a smaller intercommutation probability than ordinary cosmic strings [6]. In this paper we consider whether the mechanism of formation of cosmic superstrings might provide another source of distinction, by studying in detail the formation of string and brane defects, emphasizing differences with the standard picture of defect formation.

To set the stage, let us briefly recall how defect formation works in a standard theory [7] (see, for example [8], for modern review). Consider a scalar field theory with potential $\frac{\lambda}{4}(|\phi|^2-\eta^2)^2$. In the standard picture as the universe cools below some critical temperature T_c the U(1) symmetry is broken and ϕ rolls to the degenerate vacua of the theory $|\phi| = \eta$. Causality implies that the field cannot be correlated throughout the space and hence the field rolls to different vacua in different spatial regions leading to the formation of topological defects. The defect separation is set by the correlation length, ξ. For a universe expanding with Hubble rate H one expects $\xi < H^{-1}$ and hence a minimum defect density of about one per Hubble volume. However, this is only an upper bound on ξ. A more careful estimate can be made using condensed matter physics methods: equating the free energy gained by symmetry breaking with the gradient energy lost. Very close to T_c thermal fluctuations which can restore the symmetry are probable; however, once the universe cools below the Ginsburg temperature, T_G, there is insufficient thermal energy to excite a correlation volume into the state $\phi = 0$ and the defects "freeze out." For the scalar field theory described above this estimate yields a correlation length of order the microscopic scale: $\xi \sim \lambda^{-1}\eta^{-1}$.

The physics of tachyon condensation on the unstable Dp brane-antibrane system is quite different however, due to the peculiar form of the action [9]

$$S = -2\tau_p \int d^{p+1}x\, e^{-|T|^2/b^2} \sqrt{1+|\partial T|^2}\,. \tag{1.1}$$

The tachyon potential has a "runaway" form and there are no oscillations of the field near the true vacuum which can restore the symmetry. The decaying exponential potential of the complex tachyon field multiplies the kinetic term. Once condensation starts, T rolls quickly to large value, and damps the gradient energy exponentially. This essentially eliminates the restoring force which would tend to erase gradients within a causal volume in an ordinary field theory.

It is of particular importance to determine the correlation length for the defect network at the end of brane-antibrane inflation since this will determine what kinds of defects are formed. If the correlation length is much larger than the string scale, as previous studies have suggested [1], then the D$(p-2)$-branes formed in the annihilation of the Dp-D$\bar{p}$ pair which drive inflation will always wrap the same compact directions as the parent branes and hence only cosmic-string-like defects will be produced. This is because the extra dimensions are typically compactified near the string scale and if $\xi \gg l_s$ then the tachyon field will always be correlated in the extra dimensions. On the other hand, if $\xi \sim l_s$, then defects may be formed which are localized in the extra dimensions and may appear as textures, domain walls or monopoles to the 4-dimensional observer, depending on which compact dimensions the parent branes wrap. Domain walls are cosmologically dangerous and will overclose the universe, hence if $\xi \sim l_s$ then Dp-D$\bar{p}$ inflation where $p > 3$ may be ruled out.

JHEP06(2005)075

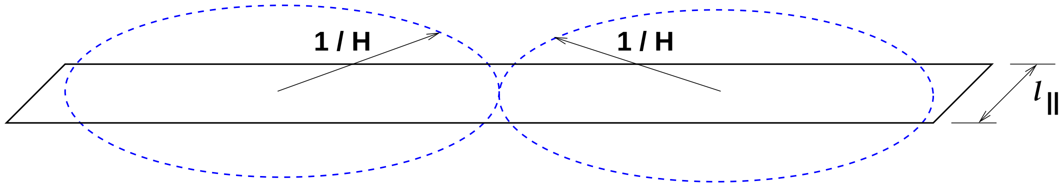

Figure 1: Illustration of the Kibble argument in the case of branes spanning extended dimensions and wrapping compact ones. The compact dimension is much smaller in size than the Hubble distance, $1/H \gg \ell_{\|}$, so there are no causally disconnected regions along this dimension. The branes that will form as topological defects will wrap the same compact dimensions as the parent branes, and will be localized on the extended dimensions.

In this paper we perform a quantitative analysis of the formation of string defects starting from the unstable tachyonic condensate that describes unstable brane-antibrane systems. Having established that defects form with an energy density comparable to that which is available from the condensate, we then examine the possible cosmological consequences of this larger-than-expected initial density and of the possibility of producing textures, domain walls and monopoles rather than just cosmic strings.

The organization of this paper is as follows. In section 2 we briefly discuss brane-antibrane inflation and the formation of defects at the end of brane-antibrane inflation by tachyon condensation. In section 3 we study the formation of codimension-one branes in the decay of a nonBPS brane both analytically as well as through lattice simulations. In section 4 we repeat the study, this time for the formation of codimension-two branes in the decay of the brane-antibrane system. We contrast these results with the formation of conventional cosmic strings in section 5. The reader who is interested in the cosmological implications may wish to skip directly to section 6 where we consider new constraints on brane inflation models which may arise due to the large initial density of defects. Conclusions are given in section 7. In an appendix we justify the assumed initial conditions for the tachyon fluctuations which lead to defects.

2. Brane inflation

Relic cosmic superstrings can form at the end of inflation driven by the attractive interaction of branes and antibranes [10]. In this picture, inflation ends when the brane and antibrane (or a pair of branes oriented at angles) become sufficiently close that one of the open string modes stretching between the branes becomes tachyonic. The subsequent rolling of this tachyon field describes the decay of the brane-antibrane pair. Quantum fluctuations produce small inhomogeneities in the tachyon field, which will cause it to roll toward different vacua in different spatial regions, leading to the formation of topological defects which are known to be consistent descriptions of lower dimensional branes.

The branes which drive inflation must span the three noncompact dimensions and may wrap some of the compact dimensions. The defects which form are lower dimensional branes whose world-volume is within the world-volume of the parent branes. The argument was made in [1] (illustrated in figure 1) that by applying the reasoning of the Kibble mechanism to the formation of the lower-dimensional branes one concludes that the branes which are copiously produced as topological defects will always wrap the same compact dimensions that the parent branes wrap and hence these branes appear as cosmic strings to the 3-dimensional observer.[1] This argument is based on the fact that the size of the compact dimensions is orders of magnitude smaller than the Hubble distance during (and at the end of) inflation and hence there are no causally disconnected regions along the compact dimensions (a reasonable estimate is $H^{-1} \sim 6 \times 10^3 M_s^{-1}$ while typically the extra dimensions are compactified at a scale R closer to M_s^{-1}). Therefore along these directions the tachyon field would always roll toward the same vacuum and no topological defects would form. The causally disconnected regions occur only along the extended directions, so the defects are localized along the extended directions. An identical argument implies that cosmic strings should form with a density of about one string per Hubble volume.

Reference [1] uses the analogue of the condensed matter physics argument paraphrased in section 1 to estimate the correlation length of the tachyon field at the end of inflation. Since the universe has zero temperature at the end of inflation the thermal fluctuations are replaced by the quantum fluctuations of de Sitter space: $H/(2\pi)$. In that analysis the potential depends on the brane separation and as a result of the relative motion of the brane and the antibrane, the curvature of the potential at $T = 0$ changes sign. It is also assumed that the tachyon potential, $V(T)$, has a minimum for some finite value of T. In this case there is an interplay between the correlation length, ξ_C given by the curvature of the potential at $T = 0$, and the Ginsburg length, ξ_G, which is the largest scale on which quantum fluctuations of the field can restore the symmetry. As the brane separation decreases and the shape of the potential changes, the correlation length and the Ginsburg length change as well. The density of defects freezes when $\xi_C = \xi_G$. This analysis yields a correlation length which is not much different from the Hubble scale: $\xi \sim 1.6 \times 10^4 M_s^{-1} \sim 2.7 H^{-1}$.

There are several reasons to consider a more quantitative study of defect formation at the end of brane inflation. Firstly, the estimate of one defect per Hubble volume provides only a lower bound on the defect density via causality. The actual network of defects produced is determined by the complicated tachyon dynamics as it rolls from the unstable maximum of its potential to the degenerate vacua of the theory. Second, the effective field theory which describes the dynamics of the tachyon [9] has a rather unusual causal structure [11] and the usual reasoning of the Kibble mechanism may not be applicable. In [11] it was found that in the case of the homogeneous rolling tachyon the small fluctuations of the field propagate according to a "tachyon effective metric" which depends on the rolling tachyon background. As the tachyon rolls towards the vacuum the tachyon effective metric degenerates, the tachyon light cone collapses onto a time-like half line and the tachyon fields at different spatial points are causally decoupled.

Finally we comment on the validity of the effective field theory description. At some point the effective field theory description of the decaying brane will become inappropriate

[1]The production of phenomenologically dangerous monopole-like and domain wall-like defects is suppressed.

and the correct degrees of freedom will be the decay products of the brane annihilation. Since the topological defects of the tachyon field form in a short time of order M_s^{-1}, [12]-[14], the effective field theory description should be applicable during the period of defect formation. Furthermore, there are reasons to believe that the effective field theory is valid even at late times when the tachyon is close to the vacuum, since in this regime the tachyon effective field theory may give a dual description of the closed strings which are produced during brane decay [15].

3. Tachyon kink formation in compact spaces

In this section we consider the formation of codimension-one branes from tachyon condensation on a nonBPS brane. We study defect formation both by solving the full nonlinear equations of motion on a lattice, as well as providing analytical approximations to the behavior of the tachyon field in different regimes of interest. We study the evolution of the tachyon field starting from a profile which is close to the unstable vacuum $T = 0$ (in this respect our analysis is similar to [16, 17]). In our analysis there is no parameter which causes continuous variation of the potential, we consider the brane-antibrane system to be coincident at $t = 0$ when the initial conditions are imposed (this potentially important difference should be kept in mind when comparing our results to [1]).

3.1 Action and equation of motion

We will work with the tachyon Dirac-Born-Infeld effective action [9, 18, 19]

$$S = -\int V(T)\sqrt{1+\partial_\mu T\partial^\mu T}\,\sqrt{-g}\,d^{3+1+1}x\,. \tag{3.1}$$

The action (3.1), with $V(T) = V_0\,/\cosh(T/\sqrt{2\alpha'})$, can be derived from string theory in some limit [20] and has been shown to reproduce various nontrivial aspects of the full string theory dynamics [19, 21]. Here we take the potential to be

$$V(T) = \tau_p\,\exp\left(-T^2/\,b^2\right) \tag{3.2}$$

where τ_p is the Dp-brane tension. The constant b determines the tachyon mass in the perturbative vacuum as $M_T = \sqrt{2}b^{-1} \sim 1/\sqrt{\alpha'}$. Qualitatively the results will depend very little on the specific functional form of $V(T)$. In fact, for much of the analytical analysis which follows we will not even need to make reference to the specific functional form of $V(T)$.[2] We consider real tachyon fields in a spacetime with three expanding, noncompact dimensions $\{x^i\}$ (where $i = 2, 3, 4$) and one compact dimension $x^1 = x$ which we take to be static. The metric is

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + a(t)^2\delta_{ij}\,dx^i\,dx^j\,. \tag{3.3}$$

For simplicity we take the tachyon field to depend only on time, $x^0 = t$, and the compact spatial coordinate $x^1 = x$. The equation of motion which follows from (3.1) with the

[2] Provided of course that $V'(T = 0) = 0$ and $V(T \to \pm\infty) \to 0$.

metric (3.3) is

$$\left(1+T'^2\right)\ddot{T} = T''\left(1-\dot{T}^2\right) - \left(3H\dot{T} + \frac{V'(T)}{V(T)}\right)\left(1-\dot{T}^2+T'^2\right) + 2\dot{T}\,T'\,\dot{T}' \qquad (3.4)$$

where $\dot{T} = \partial_0 T = \partial_t T$, $T' = \partial_1 T = \partial_x T$, $V'(T) = \partial V/\partial T$ and $H = \dot{a}/a$.

3.2 Lattice simulation of kink formation

We have solved (3.4) on a lattice for different values of the compactification radius and the Hubble parameter H (which we take to be constant for simplicity). We consider vanishing initial velocity $\dot{T}(t=0,x) = \dot{T}_i(x) = 0$. The initial profile $T(t=0,x) = T_i(x)$ is a Fourier series truncated such that the minimum wavelength is comparable to the lattice spacing.[3] The amplitudes of the Fourier coefficients are given by a random Gaussian distribution and the overall amplitude of the initial profile is chosen to be small compared to b.

As in [13] we find that the gradient of the tachyon field near the core of the kink becomes infinite in a finite time, forcing us to halt our evolution. After this point the codimension-one branes have formed and if one wants to follow the evolution beyond this time it is necessary to consider the dynamics between branes and antibranes in a compact space; the field theory description is no longer adequate.

Typically the initial profile crosses $T = 0$ at many points. In the early stages of the evolution the field begins to grow due to the small displacement from the unstable vacuum. During this phase of the evolution many of the small fluctuations of the field will straighten themselves out. Large Hubble damping tends to kill off the high frequency fluctuations faster. At the end of this initial stage there may be several locations where the field stays pinned at $T = 0$, depending most crucially on the radius of compactification. The evolution very quickly enters a nonlinear regime in which the field begins to roll quickly towards $T \to +\infty$ where $T > 0$ and $T \to -\infty$ where $T < 0$.

The most important parameter for determining the density of the defect network once the daughter branes have formed is the radius of compactification, R. Perhaps surprisingly, we find that for a compactification radius as small as a few times b, tachyon kinks will form. Once the field enters the nonlinear regime the defect formation depends only on the local physics near the core of the kink. In figures 2, 3 we plot the tachyon field versus t and x, showing the formation of codimension-one branes from the decay of a nonBPS brane for radii of compactification $R = 8M_T^{-1}$ and $R = 15M_T^{-1}$. The Hubble constant is taken to be vanishing, $H = 0$, in these figures. We have also considered $H \neq 0$ and find that the Hubble damping has little effect on the final kink/anti-kink network. Hence we find that tachyon kinks *do* form in the compact directions even when the field is initially in causal contact throughout the extra dimension.

3.3 Analytical study of tachyon kink formation

Here we describe analytically the formation of tachyon kinks. Since the full equation of motion (3.4) is difficult to solve analytically for arbitrary initial data, we study the

[3] See the appendix for a discussion of the validity of these initial conditions.

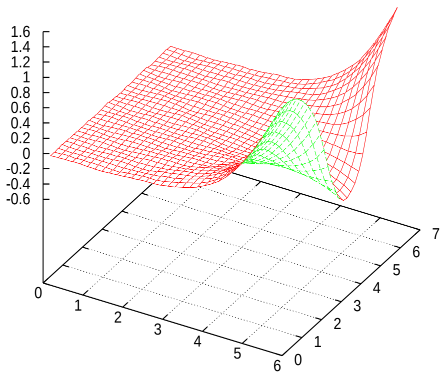

Figure 2: Formation of tachyon kinks for $R = 8M_T^{-1}$. The time axis is measured in units of M_T^{-1}.

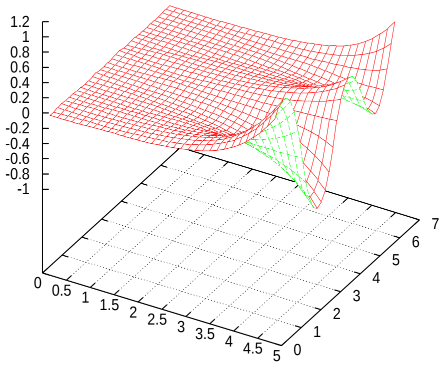

Figure 3: Formation of tachyon kinks for $R = 15M_T^{-1}$. The time axis is measured in units of M_T^{-1}.

dynamics of (3.4) in different regimes: near the core of the defect, where the field stays pinned at $T = 0$, and away from the core, where T rolls towards the vacuum. To simplify our analysis we neglect the compact topology of the extra dimension x, which should be a good approximation since the defect solutions are infinitely thin and therefore highly localized.

3.3.1 Solutions near the core of the defect

Here we briefly review the analytical studies of kink formation near the core of the defect presented in [14]. Consider initial data $T(t = 0, x) = T_i(x)$ and $\dot{T}(t = 0, x) = \dot{T}_i(x) = 0$. The field should start to roll where $T_i(x) \neq 0$ due to the small displacement from the unstable maximum $V'(T) = 0$, and furthermore it must stay pinned at $T = 0$ at the core of the kink. At $t = 0$ the equation of motion (3.4) is

$$\ddot{T}_i(x)\left(1 + T_i'(x)^2\right) = T_i''(x) + \frac{2}{b^2} T_i(x)\left(1 + T_i'(x)^2\right).$$

Clearly any point x_0 where $T_i(x_0) = T_i''(x_0) = 0$ will be a fixed point where $\ddot{T}(t, x_0) = 0 = \dot{T}(t, x_0)$ throughout the evolution. We restrict ourselves to considering only initial data such that $\mathrm{sgn}(\ddot{T}_i(x)) = \mathrm{sgn}(T_i(x))$ for all x to ensure that the solutions are increasing. To analytically study the dynamics near x_0 it is therefore reasonable to make the ansatz

$$T(t, x) \cong u(t)(x - x_0). \tag{3.5}$$

For $u(t) = \text{const} \to \infty$ this solution corresponds to the singular static soliton solution of Sen [19]. Inserting the ansatz (3.5) into (3.4) and working only to linear order in $(x - x_0)$ one obtains an ordinary differential equation for the slope

$$\ddot{u} = \frac{2}{b^2} u + 2\frac{u\dot{u}^2}{1 + u^2} - 3H\dot{u}. \tag{3.6}$$

We have solved (3.6) numerically for both constant H and $H \sim 1/(t + t_0)$. We find that generically the solutions of (3.6) become singular in a finite time $t = t_c$. Larger H tends

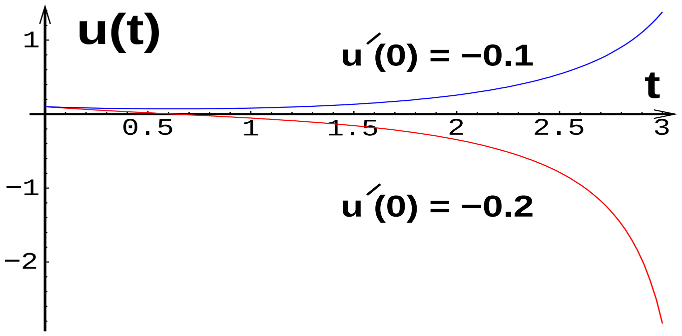

Figure 4: Behavior of the slope at the core of the kink for different initial conditions. The time axis is measured in units of b.

to delay the onset of the singularity and soften the singular behaviour. To understand this finite time slope divergence analytically it is simplest to consider $H = 0$. In this case if we assume that initially $\dot{u}$ is close to zero then the second term on the right hand side of (3.6) can be neglected and therefore

$$u(t) \cong u_+ e^{\sqrt{2}t/b} + u_- e^{-\sqrt{2}t/b}$$

at early times. Clearly $\dot{u}$ grows quickly and the second term on the right hand side of (3.6) very quickly becomes important. In the regime where u and its derivatives become large the solution has the behaviour

$$u(t) \sim \frac{k}{t_c - t}$$

where the critical time t_c depends on the initial conditions. We find that within finite time the slope becomes singular and the time dependent tachyon field near the core of the kink coincides with the singular soliton solution of Sen [19]. Hence we conclude that the codimension-one brane forms in a finite time.

This finite-time slope divergence was observed both numerically and analytically in [13] and leads to a finite-time divergence in the stress-energy tensor. This divergence was also found in an exact string theory calculation in [12]. For the potential (3.2) the stress-energy tensor near $x = x_0$ and $t = t_c$, with $H = 0$, is [14]

$$T_{00} \cong \frac{\tau_p k}{t_c - t} \exp\left(-\frac{k^2(x-x_0)^2}{b^2(t_c-t)^2}\right).$$

Hence, with the normalization $b = 2\sqrt{\pi\alpha'}$ proposed in [19], as $t \to t_c$ we have $T_{00} \to \tau_{p-1}\delta(x - x_0)$. Similarly one can show that in this regime $T_{01} = T_{11} = 0$ and that $T_{ij} = -\tau_{p-1}\delta_{ij}\delta(x - x_0)$. In the limit of condensation, then, the stress-energy near $x = x_0$ is identical to that of a D$(p-1)$-brane. If we take into account the rolling of the tachyon for $x \neq x_0$ then there will be an additional component to $T_{\mu\nu}$ corresponding to tachyon matter, as in [22].

3.3.2 Solutions near the true vacuum

Away from the site of the kink the field will roll towards the true vacuum $T \to \pm\infty$ so that $V(T) \to 0$ at late times for $x \neq x_0$. As $V(T) \to 0$ one can show both analytically [23] and numerically [24] that (3.4) has an attractor which satisfies

$$\partial^\mu T \partial_\mu T + 1 = 0 \,. \tag{3.7}$$

This attractor holds for arbitrary metric and hence the solutions we describe below hold equally well for vanishing or nonvanishing H. Exact solutions of (3.7) were found in [24] for arbitrary initial data. The solution is given along a set of characteristic curves which, in $1+1$ dimensions, are defined by

$$x(q,t) = q - \frac{T_i'(q)}{\sqrt{1+T_i'(q)^2}}\, t \tag{3.8}$$

where the parameter q defines the initial position of the curve on the x axis such that $x(q, t=0) = q$ and $T_i(x)$ is the Cauchy data at time $t=0$. The parameter q should be thought of as labeling the curves. The value of the tachyon field along a given characteristic curve is

$$T(q,t) = T_i(q) + \frac{t}{\sqrt{1+T_i'(q)^2}} \,. \tag{3.9}$$

For generic initial data these solutions form caustics in finite time where the second derivatives of the field blow up and the effective description (3.1) breaks down. In the present context the caustics lead to singularities in the stress-energy tensor [25] for $x \neq x_0$. Caustic formation does not arise in our lattice simulations since the finite-time first derivative divergence associated with defect formation forces us to halt our evolution, and this generically seems to occur before caustic formation. The formation of caustics is predicted by a wide range of tachyon effective field theories [26] and is presumably not present in the full string theory dynamics.

4. Tachyon vortex formation in compact spaces

In this section we generalize the results of section 3 to consider the formation of codimension-two branes from tachyon condensation on the brane-antibrane pair. This situation is of more direct interest to brane-antibrane inflation since inflation ends when a brane-antibrane pair annihilate. This situation is also more realistic since the stable branes in a given string theory are those whose dimension differs by a multiple of two.

4.1 Action and equations of motion

We wish to generalize the results of the previous section to consider complex tachyon fields which depend on time and two spatial coordinates which we compactify on a square torus. We generalize the action (3.1) to

$$S = -2\tau_p \int e^{-|T|^2/b^2} \sqrt{1 + \partial_\mu T \partial^\mu \bar{T}}\, \sqrt{-g}\, d^{3+2+1}x \tag{4.1}$$

with metric

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

where $x^1 = x$ and $x^2 = y$ are Cartesian coordinates on the torus and x^i denotes the three noncompact dimensions, as in section 3. We consider tachyon fields which depend on $x^0 = t$, $x^1 = x$ and $x^3 = y$. If we separate the tachyon field into into real and imaginary components as

$$T(t, x, y) = T_1(t, x, y) + i\, T_2(t, x, y)$$

then (4.1) may be rewritten in terms of components as

$$S = -2\tau_p \int \exp\left(-T_J T_J / b^2\right) \sqrt{1 + \partial_\mu T_I \partial^\mu T_I}\, \sqrt{-g}\, d^{3+2+1}x$$

where the upper-case roman indices label the real and imaginary components of the tachyon field ($I = 1, 2$). The equation of motion is (no sum over I, sum over J and K):

$$\begin{aligned} V(|T|)\, \partial_\alpha \left[\frac{\sqrt{-g} g^{\alpha\beta} \partial_\beta T_I}{\sqrt{1 + \partial_\alpha T_J \partial^\alpha T_J}}\right] &+ \frac{\partial V(|T|)}{\partial T_K} \left[\frac{\sqrt{-g} \partial_\alpha T_K \partial^\alpha T_I}{\sqrt{1 + \partial_\alpha T_J \partial^\alpha T_J}}\right] - \\ &- \frac{\partial V(|T|)}{\partial T_I} \sqrt{-g} \sqrt{1 + \partial_\alpha T_J \partial^\alpha T_J} = 0\,. \end{aligned} \tag{4.2}$$

We note that the action (4.1) has not been derived from first principles and has several drawbacks from a theoretical perspective. For other proposals of the effective field theory on the brane-antibrane pair see [19],[27]-[29]. One theoretical difficulty is that the action (4.1) does not evade Derrick's theorem [30]. This is of no practical concern to us since we study time dependent solutions and since our interest is in defect formation in a compact space where charge conservation precludes the possibility of isolated defect solutions. More seriously, for the action (4.1) a static profile of the form $T = u(x + iy)$ with $u \to \infty$ does not correctly reproduce the stress tensor for a codimension-two brane. We will attempt to address this difficulty by considering an alternative description of the complex tachyon dynamics in a subsequent subsection.

4.2 Lattice simulations of vortex formation

We solve the system of two coupled partial differential equations (4.2). As in subsection 3.2 we find that the gradient of tachyon field becomes singular near the core of the defect in a finite time, forcing us to halt our lattice evolution. As in the case of the kink we choose as initial data $T(t = 0, x, y) = \dot{T}_i(x, y) = 0$ and $T(t = 0, x, y) = T_i(x, y)$ given by a truncated Fourier series with random Gaussian coefficients with the overall amplitude small compared to b. We take $H = 0$ for our examples, since the Hubble damping plays little qualitative role in the dynamics. In figures 5, 6, 7 and 8 we plot $-|T| = -\sqrt{T_1^2 + T_2^2}$ against $\{x, y\}$ with the T axis measured in units of b. Figure 5 shows typical initial conditions used for our numerical analysis. In figures 6, 7 and 8 we plot the final configurations close to $t = t_c$ when the gradients become infinite for various radii of compactification. (Note that because we are plotting $-|T|$ rather than $+|T|$ the vortices appear as spikes in the final configuration.) Again we find that vortices do form for radii of compactification as small as a few times M_T^{-1}, even though the field is initially in causal contact throughout the compact space.

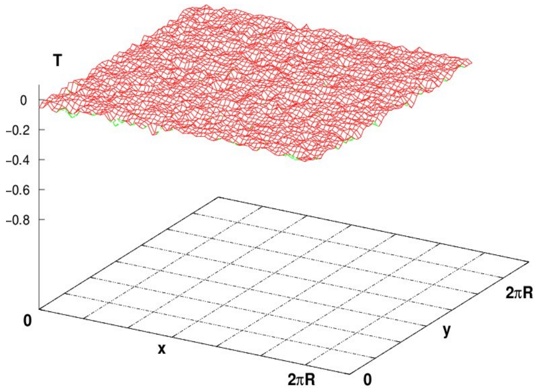

Figure 5: Typical initial configuration for numerical studies of vortex formation.

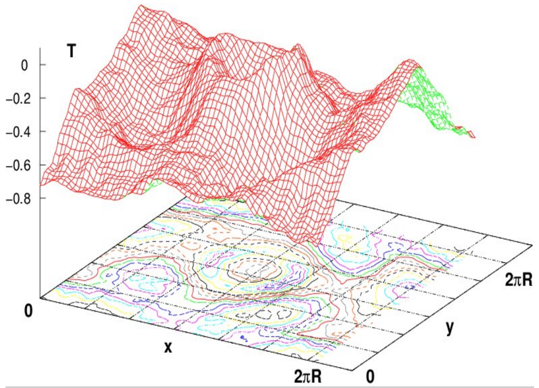

Figure 6: The final configuration of the complex tachyon field for $R = 7M_T^{-1}$.

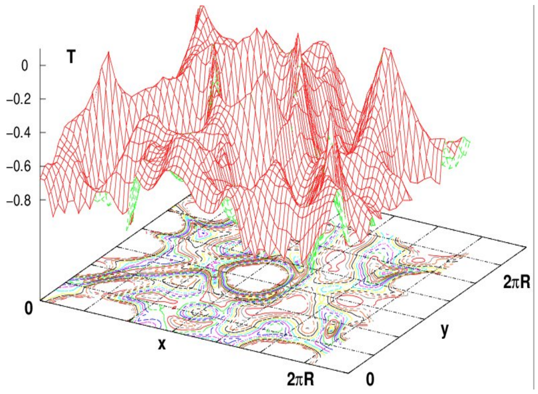

Figure 7: The final configuration of the complex tachyon field for $R = 15M_T^{-1}$.

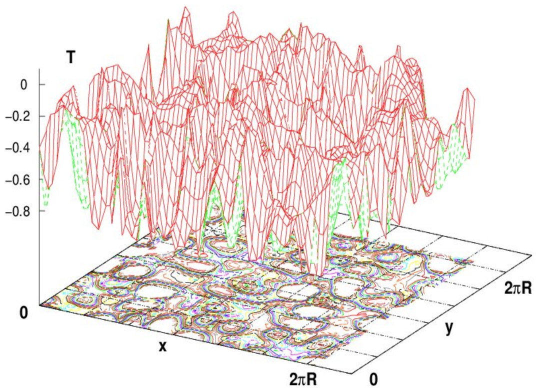

Figure 8: The final configuration of the complex tachyon field for $R = 25M_T^{-1}$.

4.3 Analytical study of vortex formation

As in subsection 3.3.1 we expect that in the case of the vortex there will exist fixed points where the field stays pinned at $T = 0$ throughout the evolution. To see this we consider the equation of motion for T_1 at $t = 0$ for initial data such that $\dot{T}(t = 0, x, y) = 0$. To simplify the expression we write the equation evaluated at a point (x_0, y_0) such that $T(t = 0, x_0, y_0) = 0$:

$$\left[-\ddot{T}_1 + \partial_x^2 T_1 + \partial_y^2 T_1\right]\left[1 + (\partial_x T_1)^2 + (\partial_y T_1)^2 + (\partial_x T_2)^2 + (\partial_y T_2)^2\right] - \\ - \Big[(\partial_x T_1)^2\, \partial_x^2 T_1 + (\partial_y T_1)^2\, \partial_y^2 T_1 + 2\, \partial_x T_1\, \partial_y T_1\, \partial_x \partial_y T_1 \partial_x T_1\, \partial_x T_2\, \partial_x^2 T_2 + \\ + \partial_y T_1\, \partial_y T_2\, \partial_y^2 T_2 + (\partial_x T_1\, \partial_y T_2 + \partial_y T_1\, \partial_x T_2)\, \partial_x \partial_y T_2\Big] = 0\,.$$

The equation for T_2 is identical with $1 \leftrightarrow 2$. It is clear, then, that any point (x_0, y_0) where $T_I(0, x_0, y_0) = \partial_x^2 T_I|_{(0,x_0,y_0)} = \partial_y^2 T_I|_{(0,x_0,y_0)} = \partial_x \partial_y T_I|_{(0,x_0,y_0)} = 0$ (for $I = 1, 2$) will be a fixed point of the evolution where $\ddot{T}_I(t, x_0, y_0) = \dot{T}_I(t, x_0, y_0) = 0$ for all t and hence the field T stays pinned at zero throughout the evolution. In the neighborhood of the point (x_0, y_0) we thus should be able to write the field in the form $T \cong u(t)(x - x_0) + v(t)(y - y_0)$

with u and v complex. Therefore to study analytically the dynamics of the field near the core of the vortex we make an ansatz of the type:

$$T_1(t,x,y) = u(t)(x-x_0), \qquad T_2(t,x,y) = u(t)(y-y_0). \tag{4.3}$$

We have chosen $v(t) = i\,u(t)$ and $u(t)$ real since the vortex solution is expected to take the form

$$T(t,z,\overline{z}) = u(t)\prod_i (z-z_i)\prod_j (\overline{z}-\overline{z}_i) \tag{4.4}$$

where we have defined complex coordinates $z = x+iy$, $\bar{z} = x-iy$. The profile (4.4), with $u(t) = \text{const} \to \infty$ was used for the multi-vortex solutions of [27] where it was shown that each holomorphic zero of (4.4) corresponds to a brane and each anti-holomorphic zero of (4.4) corresponds to an antibrane. We insert now the ansatz (4.3) into the equations of motion (4.2) which corresponds to studying the dynamics close to the core of a single vortex located at $z_0 = x_0 + iy_0$. In this regime the equations of motion for the real and imaginary parts of the field, T_1 and T_2, give the same equation for the slope near $z = z_0$:

$$\ddot{u} = \frac{2}{b^2}\,u\,(1+u^2) + \frac{3\,u\,\dot{u}^2}{1+2u^2} \tag{4.5}$$

where we have taken $H = 0$ for simplicity. We have verified numerically that this equation yields solutions which diverge in a finite time for generic initial data. In the regime where $u(t)$ and its derivatives are large the dominant contribution to (4.5) is

$$\ddot{u} \cong \frac{2u^3}{b^2} + \frac{3}{2}\frac{\dot{u}^2}{u} \tag{4.6}$$

which has the solution

$$u(t) = \frac{b}{2(t_c - t)}\,. \tag{4.7}$$

where the critical time when the slope diverges, t_c, is fixed by the initial data. This singular behavior corresponds to the finite-time formation of a codimension-two brane in the annihilation of a brane-antibrane pair.

4.4 Alternative complex tachyon action

The action (4.1) used above has the advantage of making the analysis tractable, and the resulting dynamics is analogous to kink formation. However, as discussed in subsection 4.1 this action has theoretical drawbacks. Here we consider an alternative description of the complex tachyon dynamics.

The tachyon action has been calculation in boundary string field theory (BSFT) in [28] by assuming a linear tachyon profile. For a linear profile, gauge and space-time transformations allow one to write $T = u_1 x + i\,u_2 y$, and the action one derives is

$$S = -2\tau_p \int d^{p+1}x\, \exp\left[-2\pi\alpha'\left((u_1 x)^2 + (u_2 y)^2\right)\right] F\left(4\pi\alpha'^2 u_1^2\right)\, F\left(4\pi\alpha'^2 u_2^2\right) \tag{4.8}$$

where the function $F(z)$ is given by

$$F(z) = \frac{4^z z\Gamma(z)^2}{2\Gamma(2z)} \,.$$

To make our analysis tractable we generalize the action (4.8) to nonlinear profiles using two simplifications. The first is to consider a generalization which reduces to (4.8) for linear tachyon profiles only when $u_1 = u_2 = u$. We feel this is a reasonable simplification since for the profile $T = u_1 x + i\, u_2 y$ to minimize the action one requires both u_1 and u_2 to be infinite.

Our next simplification is to replace the complicated function $F(z)$ by $\sqrt{1+\pi z}$. As justification we note that these two functions have identical behavior at large z since $F(z) \to \sqrt{\pi z}$ for $z \to \infty$.

We consider, then, the action

$$S = -2\tau_p \int \sqrt{-g}\, d^{p+1}x \exp\left(-|T|^2/\, b^2\right) \left(1 + \partial_\mu T \partial^\mu \bar{T}\right) \tag{4.9}$$

where we have performed a rescaling of T and, for consistency with (4.8), $b = \sqrt{\pi\alpha'}$. This action has also been studied in connection with tachyon condensation in [31]. Writing the tachyon field in real and imaginary components as $T(t,x,y) = T_1(t,x,y) + i\, T_2(t,x,y)$ the equation of motion one derives from (4.9) is

$$g^{\mu\nu}\nabla_\mu\nabla_\nu T_I + \frac{1}{b^2}\left[T_I\left(1 + g^{\mu\nu}\nabla_\mu T_K \nabla_\nu T_K\right) - 2T_K g^{\mu\nu}\nabla_\mu T_K \nabla_\nu T_I\right] = 0 \,. \tag{4.10}$$

We have solved the system of two coupled partial differential equations (4.10) on a lattice, as in subsection 4.2. The results are similar to those following from the action (4.1), though in this case the nonlinear effects are less dramatic. One still has defect formation, though on a longer time scale and defects begin to form for somewhat larger radius of compactification ($R \sim 30 M_T^{-1}$ is sufficient). The qualitative result that cosmic strings can form even for compactification scales well below the Hubble scale is unchanged. We consider now the dynamics near the core of the defect by plugging the ansatz

$$T_1(t,x,y) = u(t)(x - x_0)\,, \qquad T_2(t,x,y) = u(t)(y - y_0)$$

into (4.10) and working only to leading order in $(x - x_0)$, $(y - y_0)$. For $H = 0$ the equation for the slope of the field at the core of the kink is

$$\ddot{u} - \frac{1}{b^2} u = 0 \tag{4.11}$$

leading to an exponentially increasing slope. We see that in this case the slope does not become singular in finite time, which can result in a different final density of defects.

5. Comparison with ordinary cosmic string formation

If the potential for the tachyon has the usual runaway form, $\exp\left(-T^2/b^2\right)$, then once the field starts rolling, it continues rolling towards $T = \pm\infty$. The gradient force is insufficient

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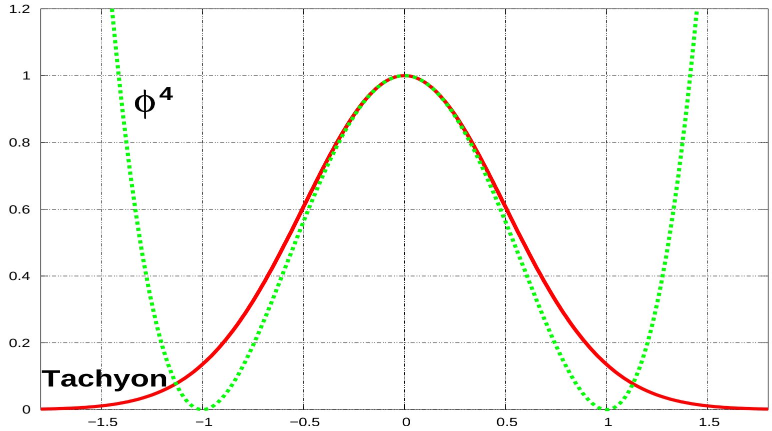

Figure 9: Comparison of the tachyon and ϕ^4 potentials. In the case of the ϕ^4 theory, the finiteness of the slope of the kink, as well as large oscillations of the field, strongly suppress the formation of defects.

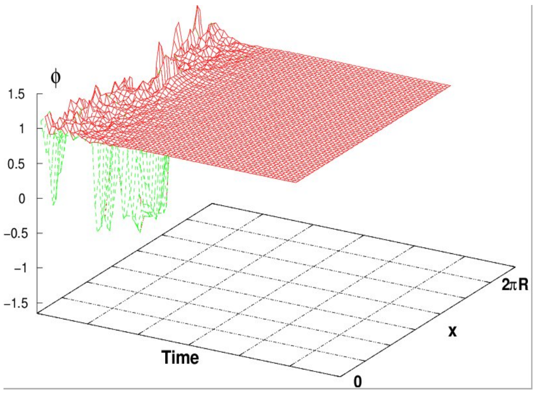

Figure 10: The effect of damping on the formation of vortices in the ϕ^4 theory. Small damping results in a large number of oscillations of the field, and effective homogenization.

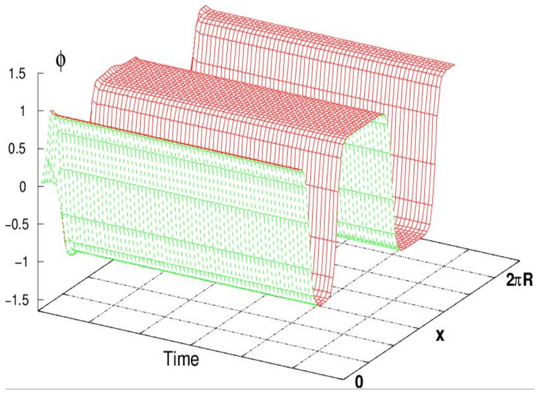

Figure 11: The effect of damping on the formation of vortices in the ϕ^4 theory. An unphysically large damping is used in order to show that the density of defects which survive is larger.

to stop or reverse the rolling. In this section we remind the reader how this differs from the mechanism for formation of defects in conventional field theories, where their production is much more strongly suppressed. The conventional case corresponds to a theory with a global U(1) symmetry, the potential $\frac{\lambda}{4}\left(|\phi|^2-\sigma^2\right)^2$ and a standard kinetic term.

In the case of ϕ^4 theory, the evolution is well-behaved and one can follow the formation and the annihilation of kinks and anti-kinks indefinitely into the future. Comparing the two potentials $\exp\left(-T^2/b^2\right)$ and $\frac{\lambda}{4}\left(|\phi|^2-\sigma^2\right)^2$ (see figure 9), we see that in the ϕ^4 theory one expects oscillations of the field which can restore symmetry and wipe out the defects.

In that case the final density of defects formed depends strongly on how fast the oscillations are damped, either through Hubble expansion, or through the coupling of the field ϕ to other fields. In figures 10 and 11 we show the effect of the damping in the ϕ^4 theory for radius of compactification $R = 60M_s^{-1}$. A large value of H results not only in a higher density of defects, but also slows down the motion of the defects that have formed, effectively reducing the rate at which they annihilate. This is in contrast to the tachyon field theory, in which case the Hubble damping plays essentially no role in determining the final density of defects.

6. Cosmological consequences

6.1 String defects

Our studies of defect formation in compact space can easily be extrapolated to imply a density of roughly one defect per string volume in the non-compact directions aswell. We now turn to the cosmological implications of having a very large initial density of strings. In normal cosmic string networks, the details of the initial conditions are not important because the network quickly reaches a scaling solution. This can be demonstrated quite simply, using the "one-scale" model for the energy density ρ in long strings whose characteristic length is L [32]:

$$\dot{\rho} \cong -2H\rho - f(P)\frac{\rho}{L}\,. \tag{6.1}$$

The terms on the r.h.s. represent respectively the effects of dilution through expansion and the loss due to breaking off small loops, which eventually disappear by shrinking and emitting gravity waves. $f(P)$ is a function of the intercommutation probability, which is believed to go like $f \sim \sqrt{P}$ [33, 34]. Taking $\rho = \mu/L^2$ (where μ is the string tension) and $H = \beta/t$, one can verify that this has a stable attractor solution

$$L = \gamma(t)\, t \equiv \frac{f(P)\, t}{2(1-\beta)} \tag{6.2}$$

known as the scaling solution, since the string length becomes a constant fraction of the horizon size H^{-1}, and the energy density in long strings also tracks that of the dominant component,

$$\rho = \frac{\mu}{\gamma^2 t^2}\,. \tag{6.3}$$

If the initial energy density was much greater than the scaling value, we can find the time scale for reaching scaling by solving (6.1) in the approximation that the Hubble expansion term is negligible compared to the loop-emission term, giving the solution $\gamma(t) = \gamma_0 + \frac{1}{2} f(P)\, \ln(t/t_0)$. Inverting, we find that the time required to reach a value $\gamma = f/(2-2\beta)$, starting from high densities where $\gamma_0 \ll 1$, is

$$t \cong t_0\, e^{1/(1-\beta)} \tag{6.4}$$

which is not much greater than t_0. For a radiation dominated universe, with $\beta = 1/2$, the scaling solution is reached in $e^2/2$ Hubble times in the usual case, and subsequent evolution is quite insensitive to the initial conditions. This conclusion is unchanged even for very small intercommutation probabilities.

We have investigated the approach to the scaling solution using a more detailed model of network evolution, which takes into account the possibility that loops may reconnect to long strings when the initial density is very high, and thereby possibly delay the onset of

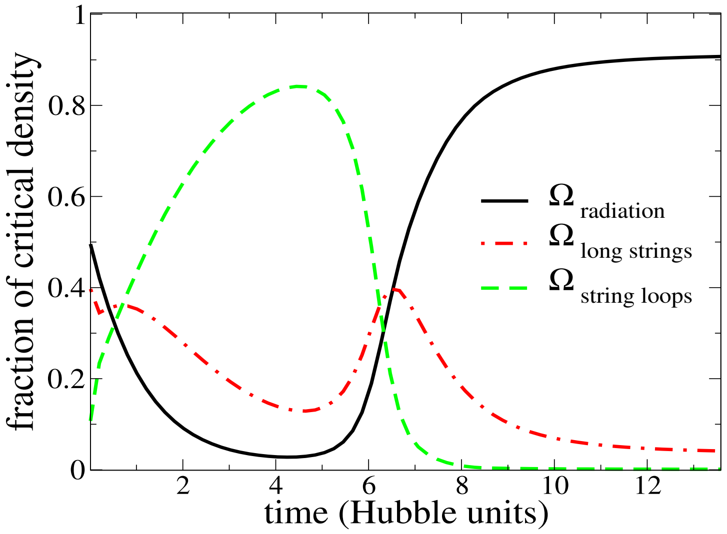

Figure 12: Fraction of critical density versus time for long strings, loops, and visible radiation starting from initial values $\Omega_s = 0.4$, $\Omega_l = 0.1$ and $\Omega_{rad} = 0.5$. Not shown is the fraction of energy density in gravity waves produced by decay of the loops.

scaling. Suppose that the density of small loops with characteristic size l is $1/x^3$, defining the average distance between loops at a given time. One can estimate that the rate per unit volume for loops to recombine with long strings is

$$\frac{dn_s}{dt} = \tilde{f}\, x^{-3}\, L^{-3}\, l\, L^2\, \frac{v_{\rm rel}}{\min(x, L)}\,. \tag{6.5}$$

Here $\tilde{f}$ is the probability of a reconnection, x^{-3} and L^{-3} are the number densities of loops and long strings, respectively, $v_{\rm rel}$ is the relative velocity between loops and strings, which we take to be $O(1)$, and $\min(x, L)$ is the distance a loop typically travels before reaching a string. The probability of a collision must be proportional to l, not l^2, since the size of the loop in the direction parallel to the long string does not affect the cross section.

In this model, long strings and small loops are treated as two separate components, $\rho_s = \mu/L^2$ and $\rho_l = \mu l/x^3$, whose energy densities are governed by

$$\begin{aligned} \dot{\rho}_s &= -2\, H\rho_s - f\,\frac{\rho}{L} + \mu\, l\,\frac{dn_r}{dt}\,, \\ \dot{\rho}_l &= -3\, H\rho_l + f\,\frac{\rho}{L} - \mu\, l\,\frac{dn_r}{dt} - \Gamma G\mu^2\frac{1}{x^3}\,. \end{aligned} \tag{6.6}$$

The last term represents the power emitted by loops due to gravitational radiation, $\Gamma G\mu^2$ [36], where $\Gamma \cong 50$ and $10^{-11} \lesssim G\mu \lesssim 10^{-6}$ [1]-[3]. The loop size is taken to always be a fixed fraction of the long string correlation length: $l/L = \Gamma G\mu$, so long as this is not smaller than the fundamental string length scale l_s. We have integrated these equations numerically, together with the Friedmann equation and the evolution equations for energy density in visible and gravitational radiation, keeping the short distance cutoff l_s on the size of the loops (in fact the results do not change noticeably if we assume the loops remain as small as this cutoff). This more detailed study confirms that the scaling solution is attained in only a few Hubble times, as can be seen from the time evolution of the fractions of the critical density for each component, shown in figure 12. We note, however, that in the above analysis we assumed that string velocities remain of order unity; if there is significant freezing out of the relative string motions in the large in the large dimensions this could have significant impact on the approach to scaling [32]. We note also that the effect of friction due to particle scattering may also significantly alter this picture [35].[4]

The initial density of strings is many orders of magnitude greater than the Kibble estimate, which gave the initial correlation length L as $\sim H^{-1}$ at the moment of formation of the network; instead, the initial energy density of the network is comparable to the total energy density available, $\sim \mu^2$, so that $L \sim \mu^{-1/2}$, smaller by a factor of $\mu^{-1/2}/M_p$ than the Kibble value. Since $\rho = \mu/L^2$, the initial density is greater by a factor of M_p^2/μ than the Kibble value. However, this huge excess is so shortlived that it has no observable effects. For instance, the contribution from the early nonscaling regime to the gravity wave background is negligible due to the small size of the loops which are formed and subsequently radiate during this era. Following reference [36], one can estimate the amplitude of these gravity waves at frequency ω as being of order

$$h \sim \frac{(G\mu)^{7/6}\rho_0^{7/12}}{M_p^2\omega^{1/3}} \sim 10^{-66} \tag{6.7}$$

where ρ_0 is the present energy density of the universe. The estimate (6.7) is some 40 orders of magnitude below the sensitivity of LIGO at the frequencies of interest, $\omega \sim 100$ Hz. As for the cosmic microwave background, the wavelength of density perturbations created during the nonscaling regime of the network is too short to be relevant: initially $\lambda \sim \mu^{-1/2}$, which gets stretched to the scale of the present energy density $\lambda \sim \rho_0^{-1/4} \sim 0.1$ mm. This length scale is also too small to be relevant for the formation of primordial black holes (PBH's) since the mass contained in volume λ^3 is far below that needed for cosmologically long-lived PBH's.

6.2 Domain walls

Thus the fast approach to the scaling solution for three-dimensional string networks erases all sensitivity to the initial conditions, even though the initial density is orders of magnitude greater than for conventional cosmic strings, and this is true regardless of the reduced intercommutation probability. However, there are situations where our modified understanding of the network's initial conditions may make a dramatic difference: namely, in the case of higher dimensional defects. Let us illustrate with the example of D5-$\overline{\text{D5}}$ annihilation, where two of the dimensions are wrapped on an internal manifold with coordinates (y_1, y_2) and the remaining three dimensions span the euclidean space (x^1, x^2, x^3). The codimension-two defects which form from the annihilation are D3 branes, and these can have various orientations with respect to the world-volume of the parent branes. The choices are exemplified by the three situations:

1. extended in x_3, y_1, y_2 directions, localized in x_1, x_2: looks like D1 in 3D.

[4] We thank J.J. Blanco-Pillado and Carlos Martins for bringing this to our attention.

2. extended in x_2, x_3, y_2 directions, localized in x_1, y_1: looks like D2 in 3D.

3. extended in x_1, x_2, x_3 directions, localized in y_1, y_2: looks like D3 in 3D.

The first case looks like ordinary cosmic strings to the 3D observer since the defects have only one long direction among the three large ones. Their effects have already been discussed. Case 3 is a network of 3-branes, all of whose dimensions are large. Their tension will contribute to the effective 3D cosmological constant.[5] Case 2, illustrated in figure 13, is the interesting one because these appear as domain walls to the 3D observer, and their energy density redshifts too slowly: $\rho \sim 1/a^2$ in terms of the scale factor of the large dimensions. A single domain wall of tension $\tau_2 = \eta^3$ within our horizon would dominate the present energy density unless $\eta \lesssim 1$ MeV. The effective tension of a 3-brane wrapping one compact dimension of size R is $\tau_2 = R\tau_3 \sim \mu^{3/2}$; hence the string scale would have to be $\lesssim 1$ MeV, absurdly small.

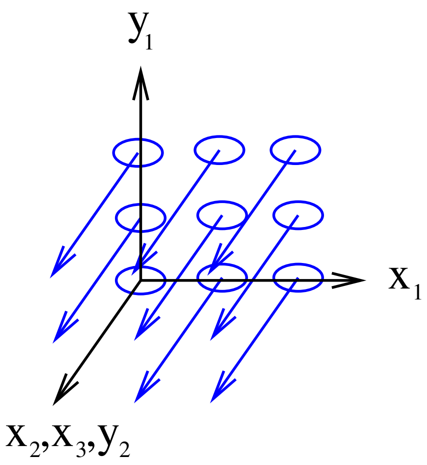

Figure 13: An array of codimension-two D3-branes from D5-anti-D5 annihilation, partially localized in the compact dimensions, which look like strings in the x_1-y_1 plane, and domain walls in the large x_i dimensions.

Our conclusion differs from that of reference [2], which speculated that no defects partially localized in compact directions can form. The argument was based on the Kibble mechanism, and thus assumed the correlation length could not be smaller than $H^{-1} \sim M_p/\mu$, whereas the size of the compact dimensions must be much smaller, of order $\mu^{-1/2}$. We have seen that in fact the initial correlation length is of the same order as the string scale, so that this argument is invalidated.

Still, one might be skeptical as to whether defects partially localized in the compact directions can survive until today, since intuitively they might be able to find each other and annihilate very quickly in the compact space. Reference [37] attempts to address this question, and concludes that domain wall defects like those in case 2 *cannot* efficiently annihilate, since they are not completely localized in the compact (nonexpanding) space. The analysis of [37] assumes that the number density of Dp-branes satisfies a rate equation which is nearly the same as that governing monopoles:

$$\dot{n} = -(3 - p_{||})Hn - \frac{\Gamma}{T^{D-2}} n^2 \tag{6.8}$$

where $p_{||}$ is the number of dimensions of the brane spanning the large dimensions, and D is the total number of spacetime dimensions. This ignores the effect of self-intersections for reducing the density of long defects, which is known to be the dominant means for string networks to reach scaling (cf. eq. (6.1)). Further, it unrealistically assumes that the defects are parallel, so that they will annihilate rather than intercommute when they meet. It is therefore not immediately clear how far we can trust their conclusions [37].

[5] These can be safely assumed to annihilate quickly since they are not separated in the expanding space (x^1, x^2, x^3).

On the other hand, numerical evolution of domain wall networks shows that self-intersections are not generic, and it is suggested that the dominant energy-loss mechanism is direct gravitational radiation rather than through collisions [38]. Furthermore it is observed that the approach to the scaling solution is slower for domain walls than for cosmic strings [39].

6.3 Full string network simulations

To attempt to address the issue of whether domain walls disappear or not, we have considered the dynamics of D3-branes in (5+1) dimensions in the approximation of projecting out the dynamics in one compact direction, y^2, and one noncompact direction, x^3, to give an effective $3+1$-dimensional system. In this case the D3-branes appear as one-dimensional objects (see figure 13) and the dynamics can be modeled by considering string evolution in an anisotropic space with two large and one small dimension. In this setup those "strings" — string-like from the (x^1, x^2, y^1) point of view — which span the large dimensions x^1 and x^2 appear as domain walls to the 3D observer while the "strings" which span the compact dimension y^1 appear as cosmic strings in 3D.

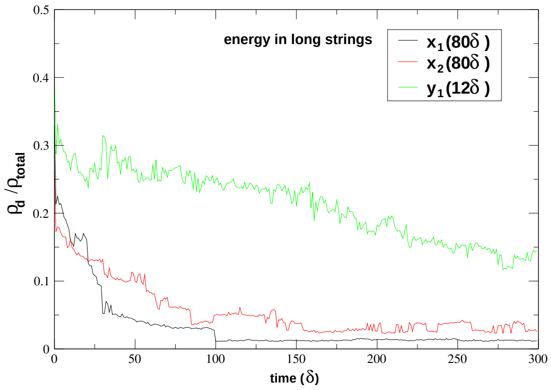

Figure 14: Fraction of wound string density about the three anisotropic directions (two large and one small) starting from equal energy density in strings wrapping each direction. Top curve is for strings winding in the small direction.

Following the setup of Smith and Vilenkin [40] we performed numerical simulations of the defect evolution in this approximation, keeping track of the extent to which defects preferentially spanned the compact direction y^1 — thus appearing as strings in the x^1, x^2, x^3 subspace, relative to spanning the large directions, which appear as domain walls. Figure 14 shows the fractional energy density in wound strings wrapped about the three directions x^1, x^2, y^1 as a function of time. This particular run started with equal energy in each direction so that the correlation lengths, and hence the interaction rates, were roughly the same. We observe that the branes wrapping the large directions lose their winding energy more quickly than those wrapping the compact direction: this can be attributed to several factors, including the smaller cross-sectional area of the long strings, and the greater energy radiated away when two long strings annihilate. The implication is that defects which look like strings to the 3D observer tend to survive preferentially over those that appear as domain walls. However since this is a toy model for the actual higher-dimensional defects, such a conclusion awaits validation from actual domain wall simulations.

As further evidence that defect evolution after formation tends to favor the defects which are localized in the large dimensions (and hence tends to favor the survival of cosmic

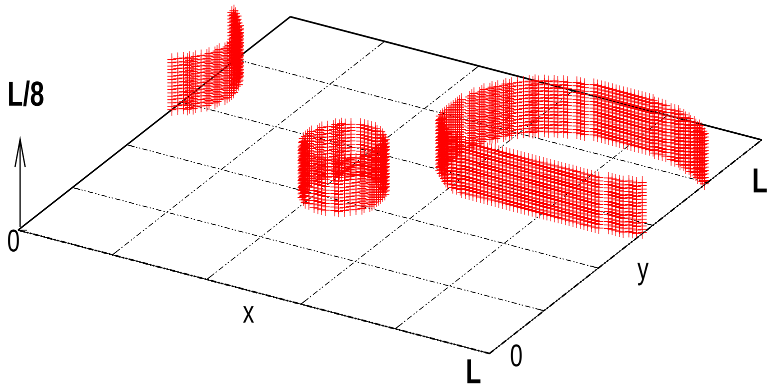

Figure 15: Plot of domain wall network in an anisotropic space with two large and one small dimensions with $L = 80M_s^{-1}$. This snapshot is taken late in the evolution and shows that domain walls which are localized in the large dimensions are preferred by the defect evolution.

string-like defects) we consider a field theory simulation of domain wall evolution for a real scalar field with potential $\frac{\lambda}{4}\left(\phi^2 - \sigma^2\right)^2$ and standard kinetic term [6]

We studied the formation and evolution of domain walls in this theory in an anisotropic $(3+1)$-dimensional space with two large and one compact dimension. Starting from a random initial profile close to the false vacuum (as in subsection 3.2) we find that the evolution of the defects after formation tends to annihilate domain walls which are localized in the compact direction and to favor the survival of domain walls which are localized in the large directions. Figure 15 shows a plot of the domain wall network late in the evolution after the domain walls localized in the compact direction have disappeared.

We also explored the effect of increasing the anisotropy, varying the initial distribution of winding modes, and varying the intercommutation probability; however, the system consistently evolved to favor winding about the compact direction. These results seem to corroborate the claim that domain wall-like defects are suppressed.

We note that there are several factors which we have not taken into account which may radically alter the picture. For example, in figure 14 we have unrealistically neglected the dynamics in two dimensions to facilitate numerical studies. Furthermore, in both of the examples above we have neglected the expansion of the large dimensions — an effect which could play an important role in the dynamics. It is therefore unclear whether domain walls will pose a cosmological problem in models of brane-antibrane inflation where the branes driving inflation wrap the compact space. We feel that this is a problem which merits further investigation and it is likely that a complete resolution of this issue will require full simulations of 3-brane dynamics in an anisotropic $(5+1)$-dimensional spacetime with 3 expanding dimensions. While such simulations would be of interest, they are beyond the scope of the present work.

[6] Though it is of more direct interest to consider the dynamics of the tachyon field theory with action (3.1) the finite time slope divergence prevents us from following the dynamics after the defects have formed.

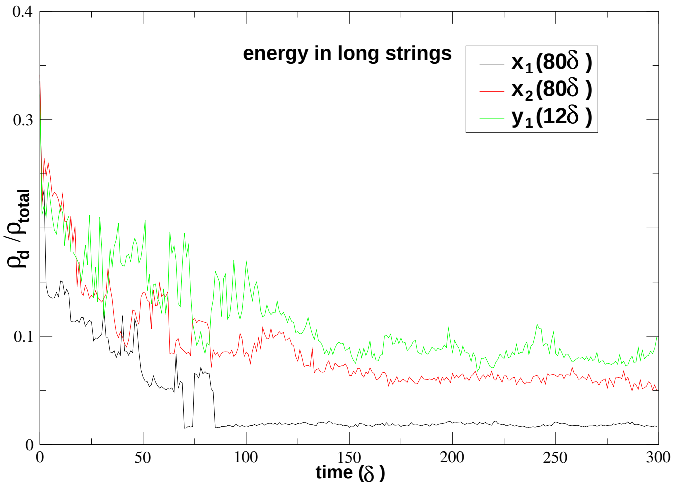

Figure 16: Fraction of wound string density about the three anisotropic directions (one large and two small) starting from equal energy density in strings wrapping each direction.

6.4 Monopoles

Finally consider an example of how monopole-like defects may be formed through a cascade of annihilations in D5-$\overline{\text{D5}}$ inflation. The initial state D5 and $\overline{\text{D5}}$ span the three large dimensions and wrap two compact dimensions. These may produce D3 and $\overline{\text{D3}}$ which wrap the two compact dimensions and are extended in one large dimension; hence these defects appear string-like to the 3-dimensional observer. A parellel D3-$\overline{\text{D3}}$ pair may then annihilate to produce D1 branes and antibranes which can span the same large dimension as the parent D3-$\overline{\text{D3}}$, or alternatively could wrap the compact dimensions. Those D1-branes which span the large dimension appear as cosmic string defects to the 3-dimensional observer while those which wrap the compact dimensions will appear as point-like (monopole) defects. If the compact dimensions admit nontrivial 1-cycles (like T^2 for example) then these monopoles will be stable. Our results indicate that in general both string-like and monopole-like defects should be produced in this cascade. To understand the subsequent evolution of such defects we consider numerically the dynamics of D1-branes in a 3D space with one large dimension and two small dimensions. Figure 16 shows the fractional energy density in wound strings wrapped about the three directions x^1, y^1, y^2 as a function of time. Strings wrapping the large direction x^1 appear as genuine cosmic strings to the 3D observer while strings wrapping small directions y^1, y^2 appear as monopole-like defects in 3D. As in subsection 6.2 we start with equal energy in each direction so that the correlation lengths are roughly equal. Again we find that strings wrapping the large direction lose their winding energy quicker than those wrapping the compact directions. Physically, this suggests that monopoles are preferentially produced over cosmic strings in this particular cascade of annihilations.

The question of whether such monopole-like defects will pose a cosmological problem depends crucially on the long-range forces between these defects and is, we feel, an issue which merits further investigation (see [41] for a solution of the monopole problem which is independent of inflation).

7. Summary and conclusions

We have studied the formation of topological defects during the decay of a nonBPS brane or a coincident brane-antibrane pair. The problem was treated both analytically, by solving the equation of motion for the tachyon field at the core of the defect, as well as numerically, by evolving the tachyon field on a lattice. We showed that defects form with a correlation length proportional to the string length rather than the horizon. Defects localized within compact dimensions can form even if the compactification radius is as small as $7M_S^{-1}$ and the tachyon dynamics is insentitive the the Hubble damping. Depending on the exact form of the action (*e.g.*, Sen's version, or the boundary string field theory version) the slope of the field at the defect could either increase exponentially in time, or else diverge within a finite time, potentially changing the initial density of defects. We compared the evolution of the tachyon field to that of a scalar field in the ϕ^4 theory and noted that the most efficient way to suppress the formation of defects is through symmetry restoration, caused by large oscillations of the scalar field. This is not possible if the potential has a runaway form, as for the string tachyon, which inevitably leads to the formation of a higher density of defects in the string case. Once the defects form the field theory description is no longer adequate, so in order to analyze the annihilation of the defects formed one has to use a description in terms of branes and antibranes interacting in a compact space.

As a result, the initial density of string defects is much greater than previous estimates. For strings which are genuine 1D objects, we showed that the string network nevertheless attains scaling behavior within just a few Hubble times, so that there are no observable consequences of the initial high string density. Of course this assumes that the network is not frustrated [42], so that scaling can indeed be achieved. Whether this is the case for cosmic superstrings which are bound states of fundamental and D-strings is an interesting open question [43].

On the other hand, we argue that in models where inflation is driven by branes which wrap the compact manifold (for example D5-$\overline{\text{D5}}$), domain wall-like and monopole-like defects are inevitably produced. The stability and subsequent evolution of such defects is complicated and may depend crucially on the details of the compactification, for example, on whether or not the compact manifold which the parent branes wrap admits nontrivial 1-cycles. We leave detailed studies of whether such models are phenomenologically viable to future investigations; however, we have shown that the formation of defects at the end of brane-antibrane inflation is much more complicated and model-dependent than one might have expected.

Acknowledgments

We would like to thank Robert Brandenberger, J.J. Blanco-Pillado, Guy Moore and Henry Tye for discussions. This work was supported in part by NSERC and FQRNT.

A. Initial conditions for defect formation

Here we briefly discuss the initial conditions for defect formation at the end of brane

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inflation. During the slow roll inflationary phase the tachyon behaves as an ordinary massive Klein-Gordon scalar field (provided $T \ll l_s^{-1}$). We consider here for simplicity a standard field theory of a scalar χ in $3+1$-dimensions whose mass squared parameter abruptly becomes negative. This type of theory has been considered in detail in [16, 17].

During the de Sitter phase (before the mass parameter becomes tachyonic) vacuum fluctuations yield a blackbody spectrum of produced particles $\langle 0|N_k|0\rangle = (e^{\omega_k/T}-1)^{-1}$ with temperature $T = H/(2\pi)$ (see, for example, [44] for a review). However, in any region of de Sitter space which is small compared to the Hubble scale, the space is locally Minkowski, and even in the vacuum state there are quantum fluctuations quantified by the two point functions of the fields

$$\begin{aligned}\langle \chi^\star(\vec{k})\chi(\vec{k}')\rangle &= \frac{1}{2|\vec{k}|}(2\pi)^3\delta^3(\vec{k}-\vec{k}') \\ \langle \Pi^\star(\vec{k})\Pi(\vec{k}')\rangle &= \frac{|\vec{k}|}{2}(2\pi)^3\delta^3(\vec{k}-\vec{k}')\end{aligned}$$

where $\Pi = \dot{\chi}$.[7] The initial stages of the string tachyon condensation are identical to the tachyonic preheating scenario described in [16, 17]. The tachyonic instability amplifies exponentially those modes with $|\vec{k}| < m$ where the χ field has mass squared parameter $-m^2$ and the variance of the fluctuations grows as [17]

$$\langle \chi^2(t)\rangle = \langle \chi^2(0)\rangle + \frac{1}{8\pi}\int_0^{m^2} dk^2 \frac{m^2}{m^2-k^2}\sinh^2\left(t\sqrt{m^2-k^2}\right)$$

where $\langle \chi^2(0)\rangle$ is a divergent vacuum contribution. The above result was derived in (3+1) dimensions, but the generalization to higher dimensions must have the form

$$\langle \chi^2(t)\rangle = \langle \chi^2(0)\rangle + c\sum_i \int_0^{m^2} dk^2 \frac{m^2}{m^2-k^2-m_i^2}\sinh^2\left(t\sqrt{m^2-k^2-m_i^2}\right)$$

where m_i are the masses of the Kaluza-Klein excitations. These fluctuations grow to be of order the classical VEV, $\langle \chi^2(t)\rangle - \langle \chi^2(0)\rangle \sim m^2/\lambda$ before the linear treatment breaks down. Notice that this growth occurs on a microscopic time scale. Since in the case of the string theory tachyon the potential has no minimum, we can conservatively take the initial exponential growth to be over a shorter time scale, $\langle \chi^2\rangle \sim m^2 \sim M_s^2$. These fluctuations are much larger than the de Sitter fluctuations $\langle \chi^2\rangle \sim H^2$ and are thus the dominant seeds for defect formation. Furthermore, these fluctuations have a minimum wavelength comparable to the string scale which is sufficient to initiate the formation of defects localized in the compact dimensions. This justifies our choice of random initial conditions for the tachyon field in the numerical studies of defect formation.

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